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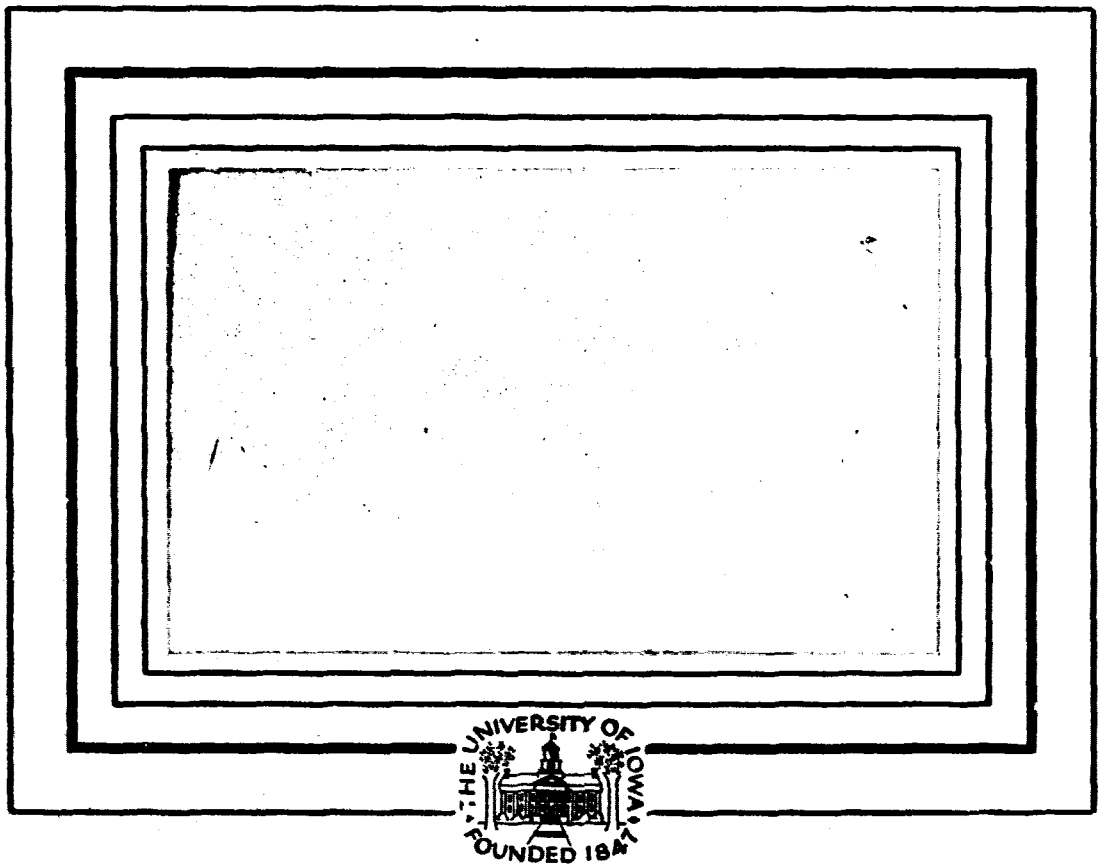

Department of Physics and Astronomy

THE UNIVERSITY OF IOWA

Iowa City, Iowa 52242

U. of Iowa 73-12

Electric Field Correlations in the Guiding-Center Plasma

by

G. Joyce,* D. Montgomery,† and M. Emery*†

Department of Physics and Astronomy
The University of Iowa
Iowa City, Iowa 52242

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†Supported in part by NASA Grant NGL-16-001-043

ABSTRACT

Electric field autocorrelations for the two-dimensional electrostatic guiding-center plasma are calculated numerically. It is concluded that the autocorrelation, averaged over a thermal equilibrium ensemble, is damped in an approximately exponential fashion, as predicted by Taylor and McNamara. Oscillatory behavior of the type predicted by Taylor and Thompson is not observed.

I. INTRODUCTION

A central quantity in the theory of the transverse diffusion in a two-dimensional electrostatic guiding-center plasma[1-10] is the autocorrelation of the $\vec{k}$th Fourier component of the electric field,

$$S_{\vec{k}}(\tau) \equiv \langle \vec{E}^{*}_{\vec{k}}(0) \cdot \vec{E}_{\vec{k}}(\tau) \rangle \quad . \tag{1}$$

The notation means the following. $\vec{E}(\vec{x}, t) = \sum_{\vec{k}} \vec{E}_{\vec{k}}(t) \exp(i\vec{k} \cdot \vec{x})$ is the electric field produced by N positively and N negatively charged rods of charge per unit length ($\pm e/\ell$). The "particles" (rods) are aligned parallel to a uniform, constant magnetic field $\vec{B} = B\hat{e}_z$ and are located by giving their x and y coordinates. End effects of the rods are neglected. The velocities of the rods are given by the local $c\vec{E} \times \vec{B}/B^2$ drift. The electrostatic field $\vec{E}$ is self-consistently determined through Poisson's equation. Periodic boundary conditions in the x and y direction are assumed, and the allowed values of $\vec{k}$ are $\vec{k} = 2\pi(n_x, n_y)/L$, where n_x, n_y are any integers, not both zero. The symbol $\langle\ \rangle$ means an ensemble average over the $\tau = 0$ positions of the particles, assuming they are distributed according to the canonical ensemble of Gibbs.

Two[1,7] theories of the temporal evolution of $S_{\vec{k}}(\tau)$ lead, making different dynamical assumptions, to strikingly different

predictions. The theory of Taylor and McNamara,[1] based on an assumption that the electric field seen by a particle obeys a jointly normal probability distribution at successive instants of time, leads to a behavior

$$S_{\vec{k}}^{TM}(\tau) = \langle |\vec{E}_{\vec{k}}|^2 \rangle \exp[-k^2 R(\tau)] \quad . \tag{2}$$

Here $R(\tau)$ is a monotonically increasing function of τ which obeys $d^2R(\tau)/d\tau^2 = -\partial V(R)/\partial R$, where $V(R)$ is given by $-(c^2/4B^2) \sum_{\vec{k}} \langle |\vec{E}_{\vec{k}}|^2 \rangle \left(1 - \exp(-2k^2R)\right)/k^2$, with $R(0) = 0$, $dR(0)/d\tau = 0$. As $t \to \infty$, $R(\tau) \to D\tau/2$, where D is, up to a factor of $\sqrt{2}$, the Taylor-McNamara diffusion coefficient.[5] On the other hand, Taylor and Thompson[7] have, by means of a variant of the random phase approximation, derived an expression

$$S_{\vec{k}}^{TT}(\tau) = \langle |\vec{E}_{\vec{k}}|^2 \rangle \cos(\Omega_{\vec{k}} \tau) \quad , \tag{3}$$

an undamped sinusoidal oscillation for which[10] $\Omega_{\vec{k}}^2 \to O(k^2)$ for k small, and is a more complicated function of k for k finite, but which is always real. The undamped oscillatory behavior of Eq. (3) contrasts sharply with the monotonically damped behavior of Eq. (2), and our purpose here is to determine via a numerical simulation of the system which, if either, prediction is correct.

II. SIMULATION

A program to follow the dynamical evolution of 4000 rods advances the particle positions by the local $\vec{E} \times \vec{B}$ drift at each time step. $\vec{E}$ is determined through Poisson's equation by means of fast Fourier transforms and particle-in-cell (P.I.C.) methods, employing area weighting.

Initial positions appropriate to thermal equilibrium are something of a problem since they are not capable of being produced by random number generators in any elementary way. We determined them from an unmagnetized two-dimensional simulation code which employed Newton's laws of motion to advance the particle positions. This program was allowed to run, starting from random initial coordinates, through a few previously-determined[11] thermal relaxation times. The spectrum $|\vec{E}_{\vec{k}}|^2$ advanced from the $1/k^2$ form characteristic of random loading to the shape shown in Fig. 1. These final coordinates were then used as initial data for the guiding-center runs.

Eight runs were carried out. Conservation of energy was maintained to three per cent throughout. Autocorrelations were computed from three separate time origins, giving us an "ensemble" of twenty-four values of $\vec{E}^*_{\vec{k}}(0) \cdot \vec{E}_{\vec{k}}(\tau)$ for each value of τ.

Typical values of $S_{\vec{k}}(\tau)$ for different $\vec{k}$ modes are shown in Figs. (2a), (2b), and (2c). The crosses are the measured experimental

points, and the curves labeled "T & M" are Eq. (2), normalized to its initial value. τ is measured in the appropriate dimensionless units.[12]

The curves in Figs. 2 are typical ensemble averages. A comparison of eight individual runs with the ensemble average of twenty-four is made in Fig. 3. (The ensemble averages are crosses.) The large fluctuations about the ensemble average are characteristic of the runs and are believed to be genuine and not due to numerical error. Such accuracy checks as we have on the program (energy conservation, independent tests of the Poisson solver, tests of the particle advancement algorithm in the analytically soluble case of two particles in a box) indicate substantially smaller accumulated errors by, say, $\tau = 20$ than the fluctuations indicated in Fig. 3. Theorems on the smallness of time-dependent fluctuation quantities about their ensemble averages are conspicuously absent in the theory of the guiding-center plasma. Persuasive theoretical arguments exist to indicate that such theorems are not to be expected, unlike the more usual case of the plasma in which no guiding-center approximation is made.

In Figs. (4a) and (4b) a comparison is made between the experimental values of $S_{\vec{k}}(\tau)$ and Eqs. (2) and (3), normalized to their initial values. Equation (2) is the curve labeled "T & M" and Eq. (3) is labeled "T & T". It is clear that while the quantitative fit of neither theory is perfect, the phenomena are correctly described qualitatively much better by Eq. (2) than by Eq. (3).

In Figs. (5a) and (5b) the behavior of the autocorrelation is shown for various $\vec{k}$-values, and a plot of $k^2R'(\tau)$ vs $k^2L^2/4\pi^2$ is given for various values of k at $\tau = 20$. Figure (5b) indicates a damping of the autocorrelation which varies correctly with increasing k^2, but indicates also a systematically weaker damping than the theory predicts.

III. DISCUSSION

We have provided what is believed to be a rather sharp experimental differentiation between two competing theories of the temporal behavior of electric field autocorrelations in a two-dimensional guiding-center plasma. We have shown the qualitative correctness of the theory of Taylor and McNamara.[1] A systematically weaker decay than predicted by theory has been observed, a phenomenon for which at present no satisfactory theoretical explanation exists. Probably it is connected with the finiteness of the discreteness parameter $(n_o\lambda_D^2)^{-1}$, since the essential approximation in reference 1 seems to be the neglect of correlations between single-particle orbits.

ACKNOWLEDGMENTS

We are indebted to Dr. F. Tappert of Bell Laboratories for use of his computer program[3] by which the differential equation for $R(\tau)$ is solved. We did not use his program to determine our $R(\tau)$, but did use it to establish the accuracy of our own. Helpful discussions with Dr. Tappert and with Dr. J. P. Christiansen are also gratefully acknowledged.

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[12]The units are discussed in detail in reference 9.

FIGURE CAPTIONS

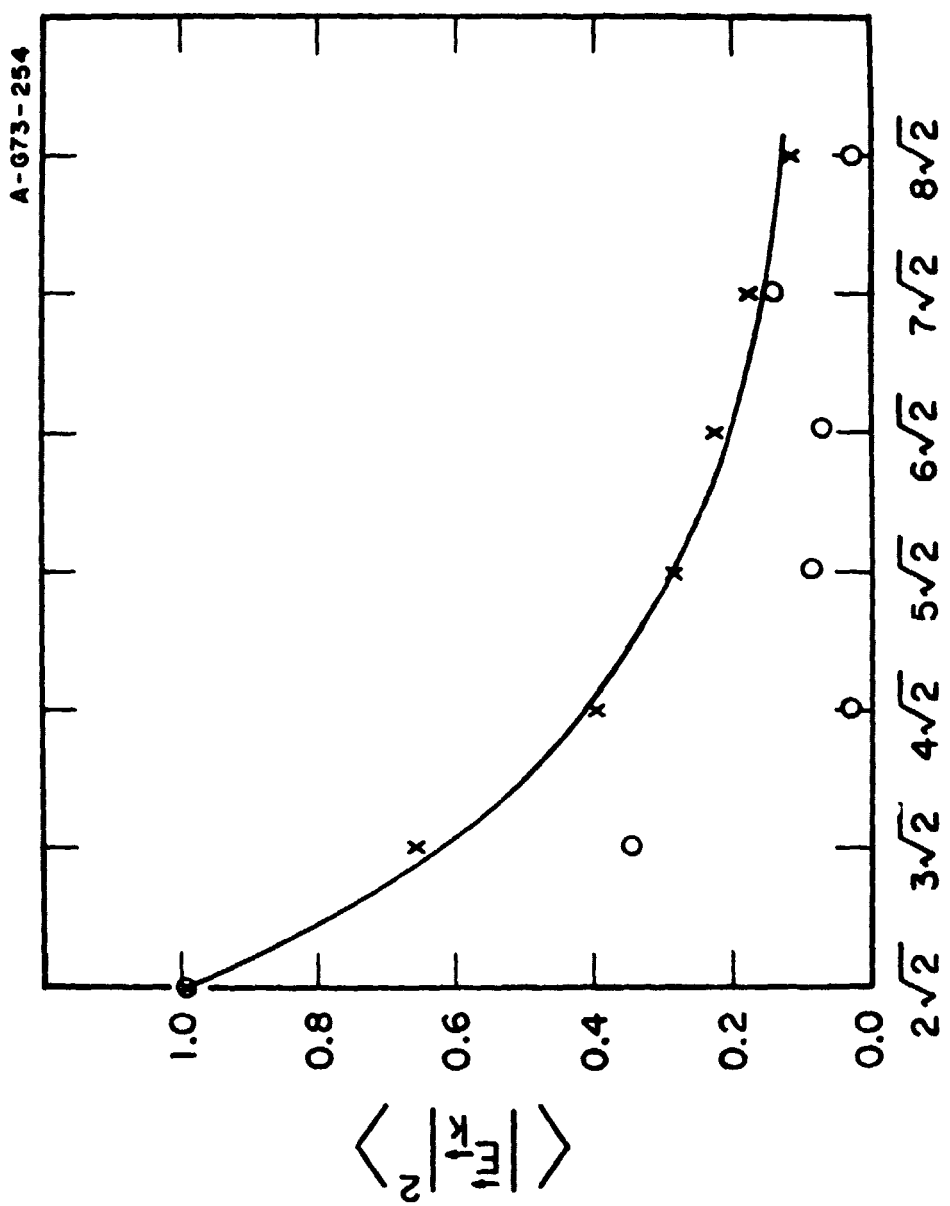
Fig. . Electric field fluctuation spectrum vs wavenumber, normalized to initial value. Solid line is theoretical thermal equilibrium curve derived from Debye-Hückel theory. Crosses are time-averaged values for an unmagnetized simulation plasma in two dimensions, where average is carried out over several relaxation times. Circles are single-time values and are present to indicate the level of typical fluctuations.

Fig. 2 Autocorrelations $S_{\vec{k}}(\tau)$ vs τ for three different modes. Crosses are measured experimental points and curve labeled "EXP" is a fit to these points. Curve labeled "T & M" is Eq. (2). Modes are labeled by values of (n_x, n_y). (2a): $(n_x, n_y) = (4, 4)$; (2b): $(n_x, n_y) = (0, 6)$; (2c): $(n_x, n_y) = (6, 6)$.

Fig. 3 Autocorrelations $S_{\vec{k}}(\tau)$ vs τ. Eight individual runs are compared with ensemble average values over a sample of twenty-four. Crosses are the ensemble average.

Fig. 4 Comparison of the results of experiment (crosses) for $S_{\vec{k}}(\tau)$ with theoretical values given by Eq. (2) (labeled "T & M") and Eq. (3) (labeled "T & T"). (4a) is for $(n_x, n_y) = (4, 4)$ and (4b) is for $(n_x, n_y) = (0, 6)$.

Fig. 5 (5a) is $S_{\vec{k}}(\tau)$ vs τ for various $\vec{k}$ values. (5b) is k^2R' ($\tau = 20$) vs $n_x^2 + n_y^2$. The solid curve is the theoretical prediction of Taylor and McNamara.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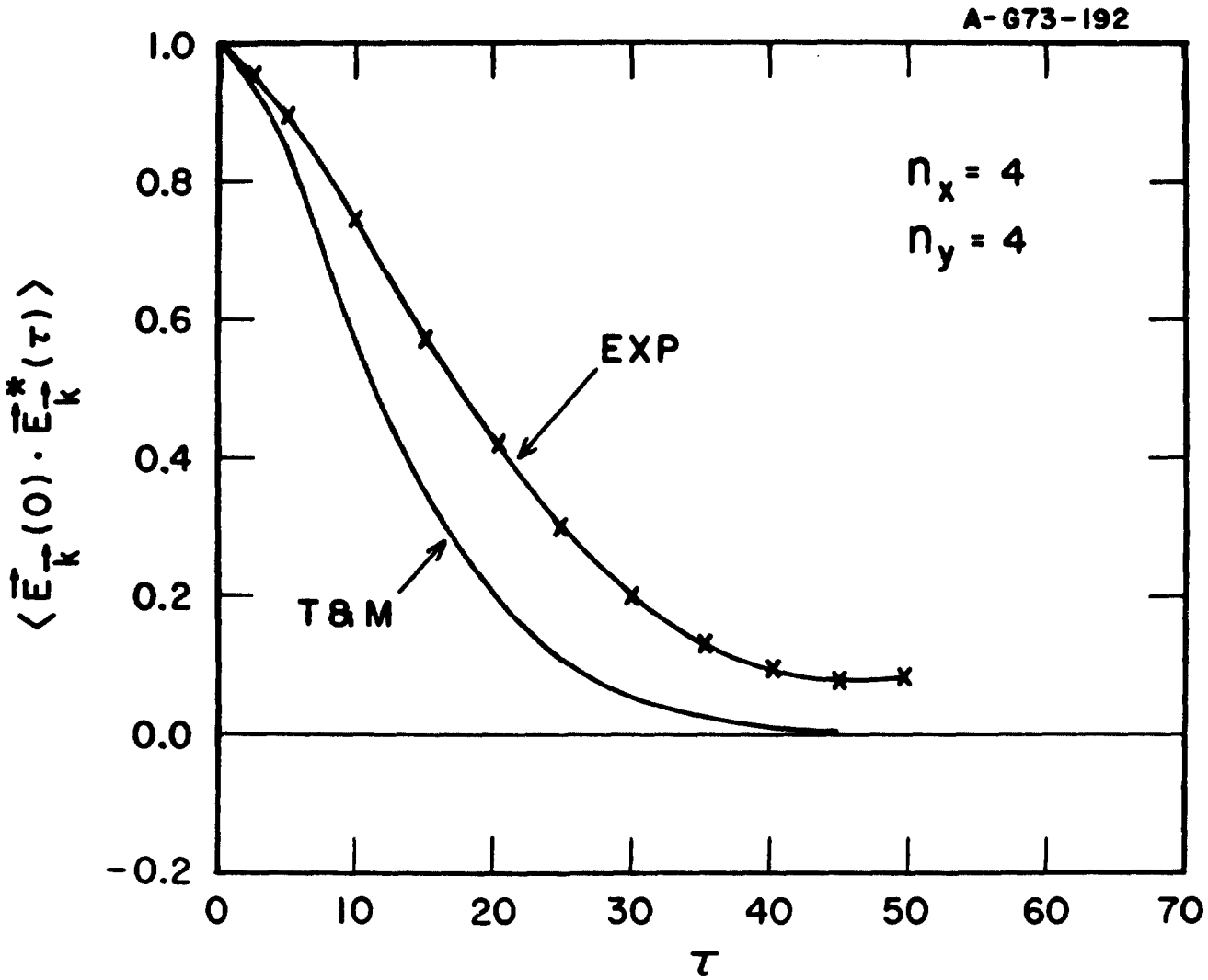
1.0
0.8
0.6
0.4
0.2
0.0
$\langle |\Gamma_{x1}|^2 \rangle$
$2\sqrt{2}$ $3\sqrt{2}$ $4\sqrt{2}$ $5\sqrt{2}$ $6\sqrt{2}$ $7\sqrt{2}$ $8\sqrt{2}$
$\sqrt{n_x^2 + n_y^2}$

Fig. 1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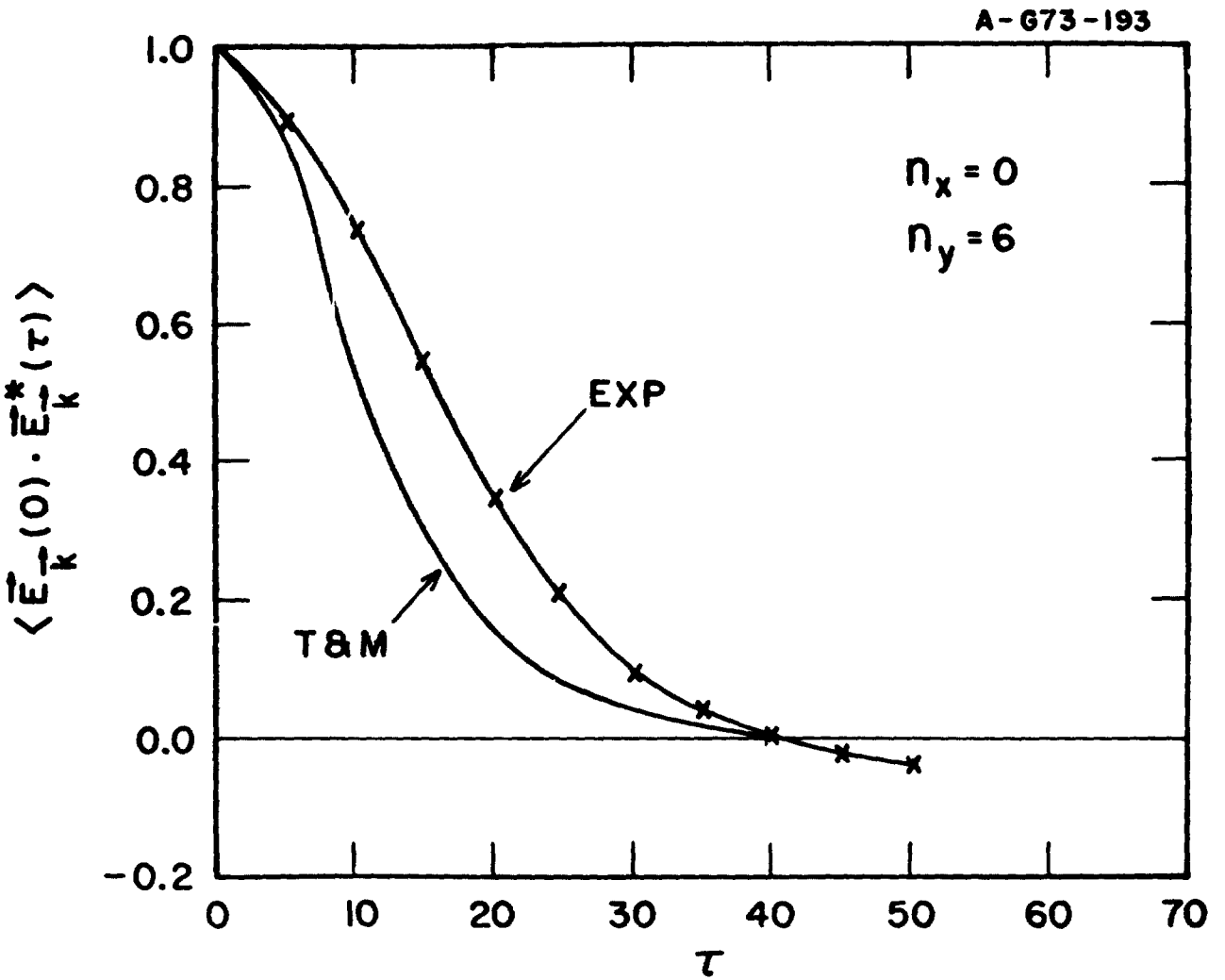
0.8
0.6
0.4
0.2
0.0
-0.2
0
10
20
30
40
50
60
70
τ
$\langle \vec{E}_{\vec{k}}(0) \cdot \vec{E}^*_{\vec{k}}(\tau) \rangle$
$n_x = 4$
$n_y = 4$
EXP
T&M

Fig. 2a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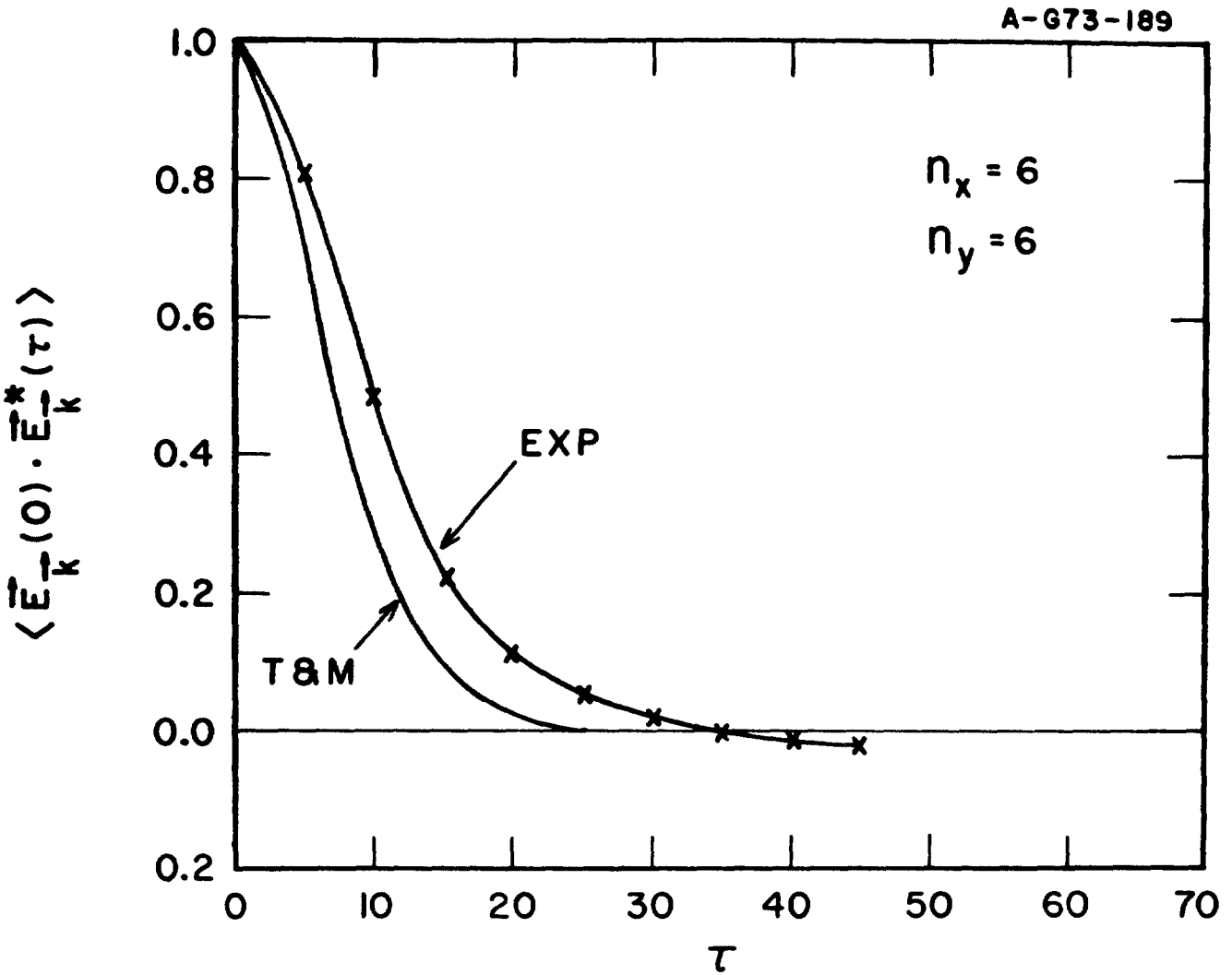
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
0
10
20
30
40
50
60
70
τ
$\langle \vec{E}_{\vec{k}}(0) \cdot \vec{E}^*_{\vec{k}}(\tau) \rangle$
$n_x = 0$
$n_y = 6$
EXP
T&M

Fig. 2b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0.8
0.6
0.4
0.2
0.0
0.2
0
10
20
30
40
50
60
70
τ
$\langle \vec{E}_{\vec{k}}(0) \cdot \vec{E}^*_{\vec{k}}(\tau) \rangle$
$n_x = 6$
$n_y = 6$
EXP
T&M

Fig. 2c

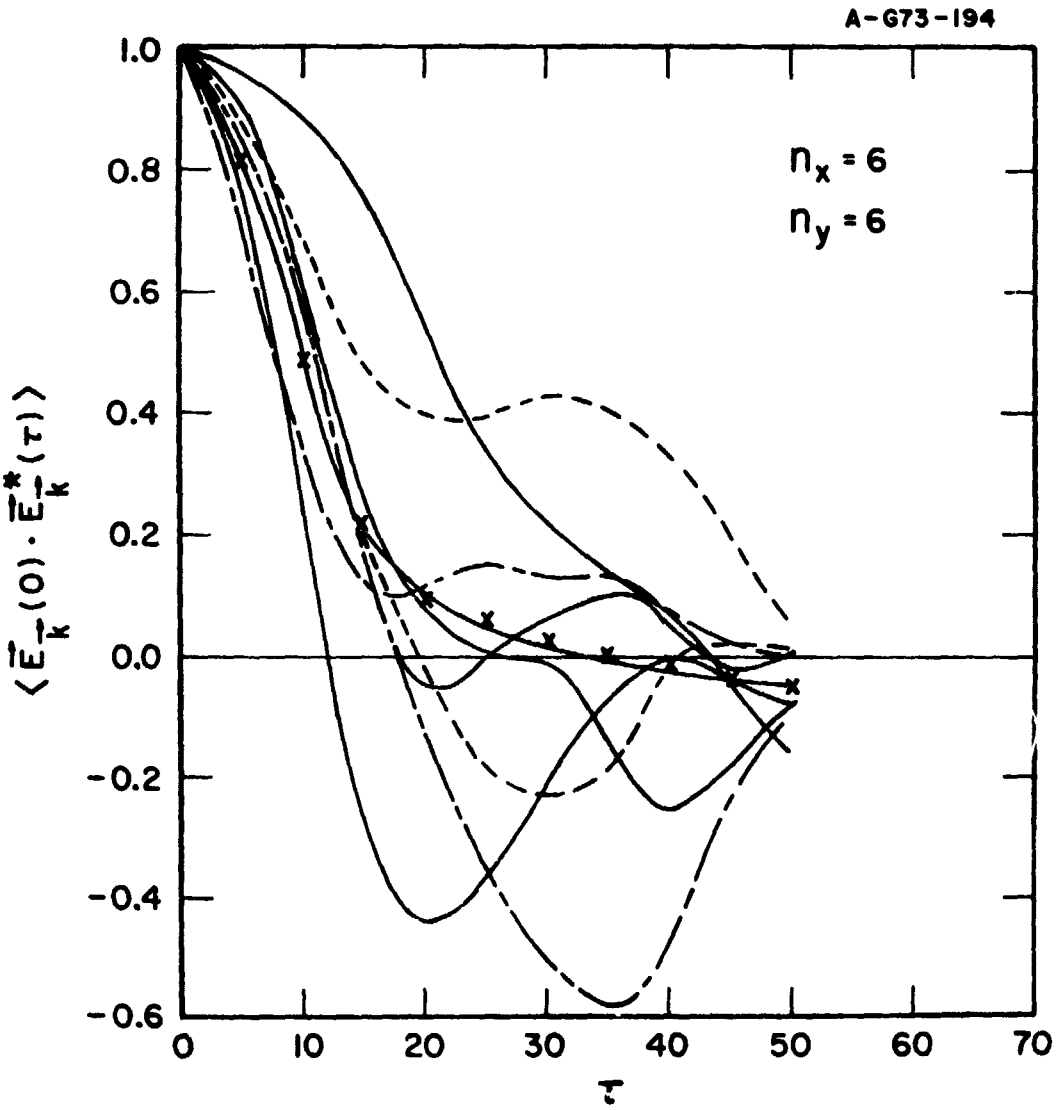
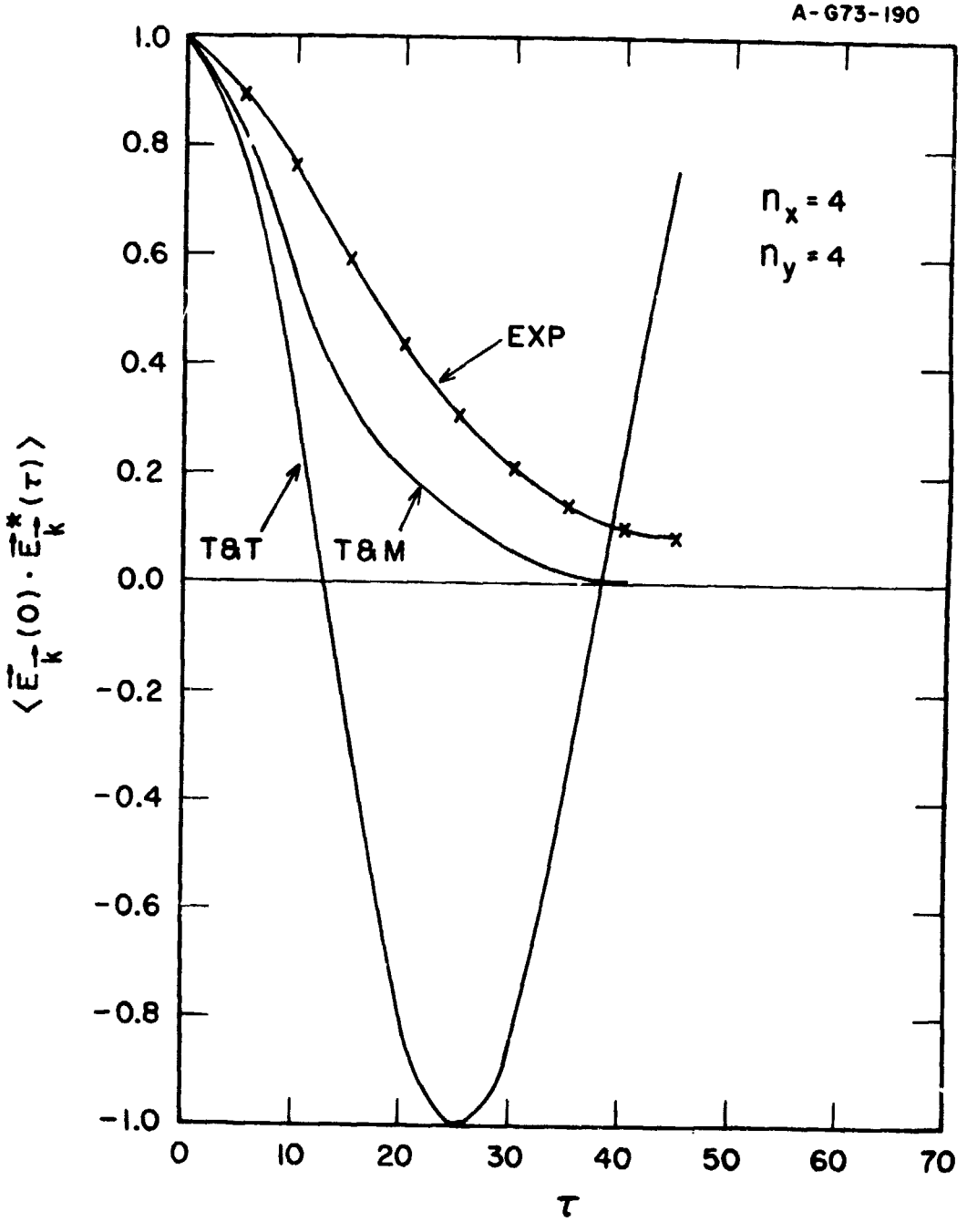

A-G73-194
$n_x = 6$
$n_y = 6$
$\langle \vec{E}_{\vec{k}}(0) \cdot \vec{E}^*_{\vec{k}}(\tau) \rangle$
τ
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
0
10
20
30
40
50
60
70

Fig. 3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0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0
10
20
30
40
50
60
70
τ
$\langle \vec{E}_{\vec{k}}(0) \cdot \vec{E}^*_{\vec{k}}(\tau) \rangle$
$n_x = 4$
$n_y = 4$
EXP
T&T
T&M

Fig. 4a

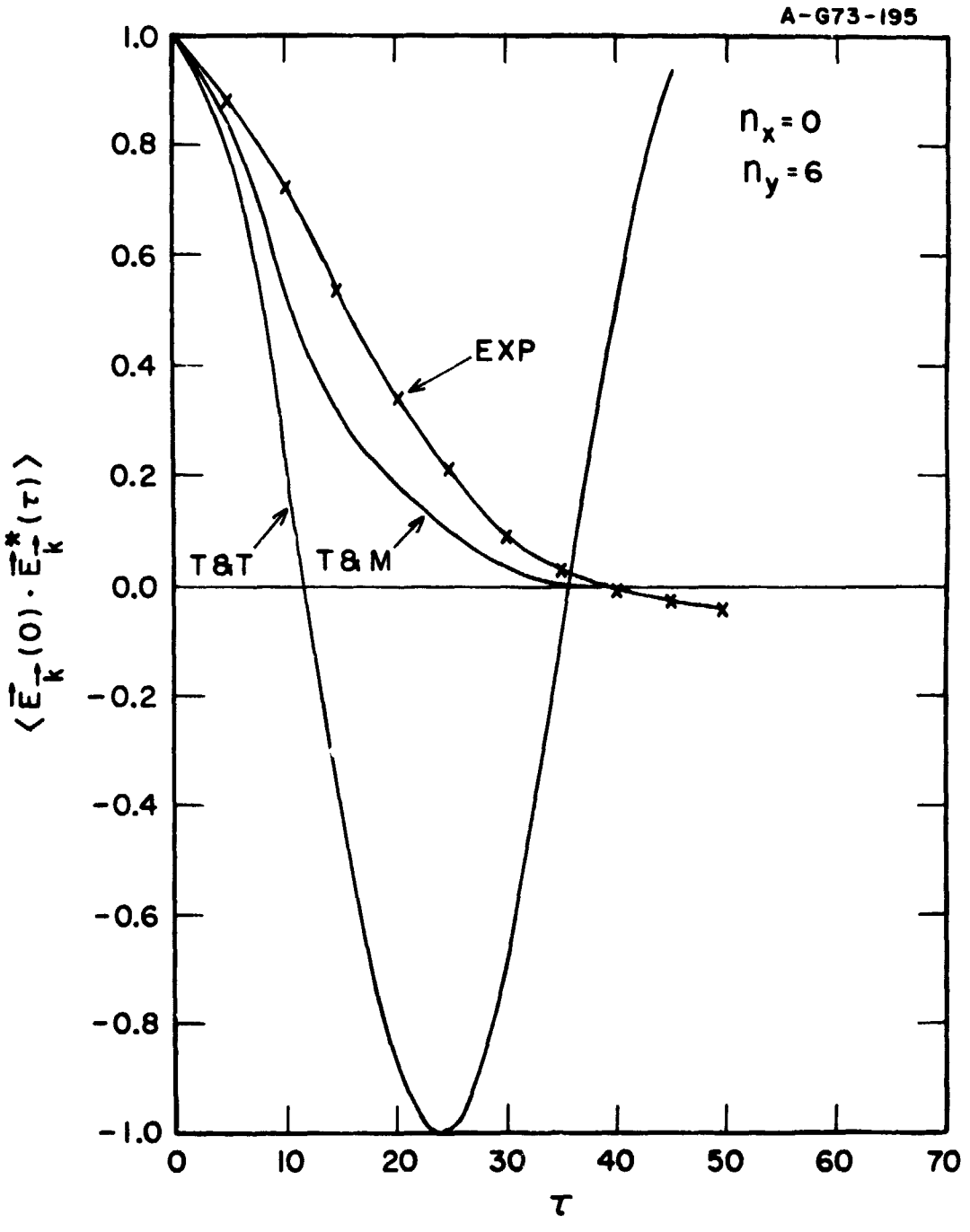
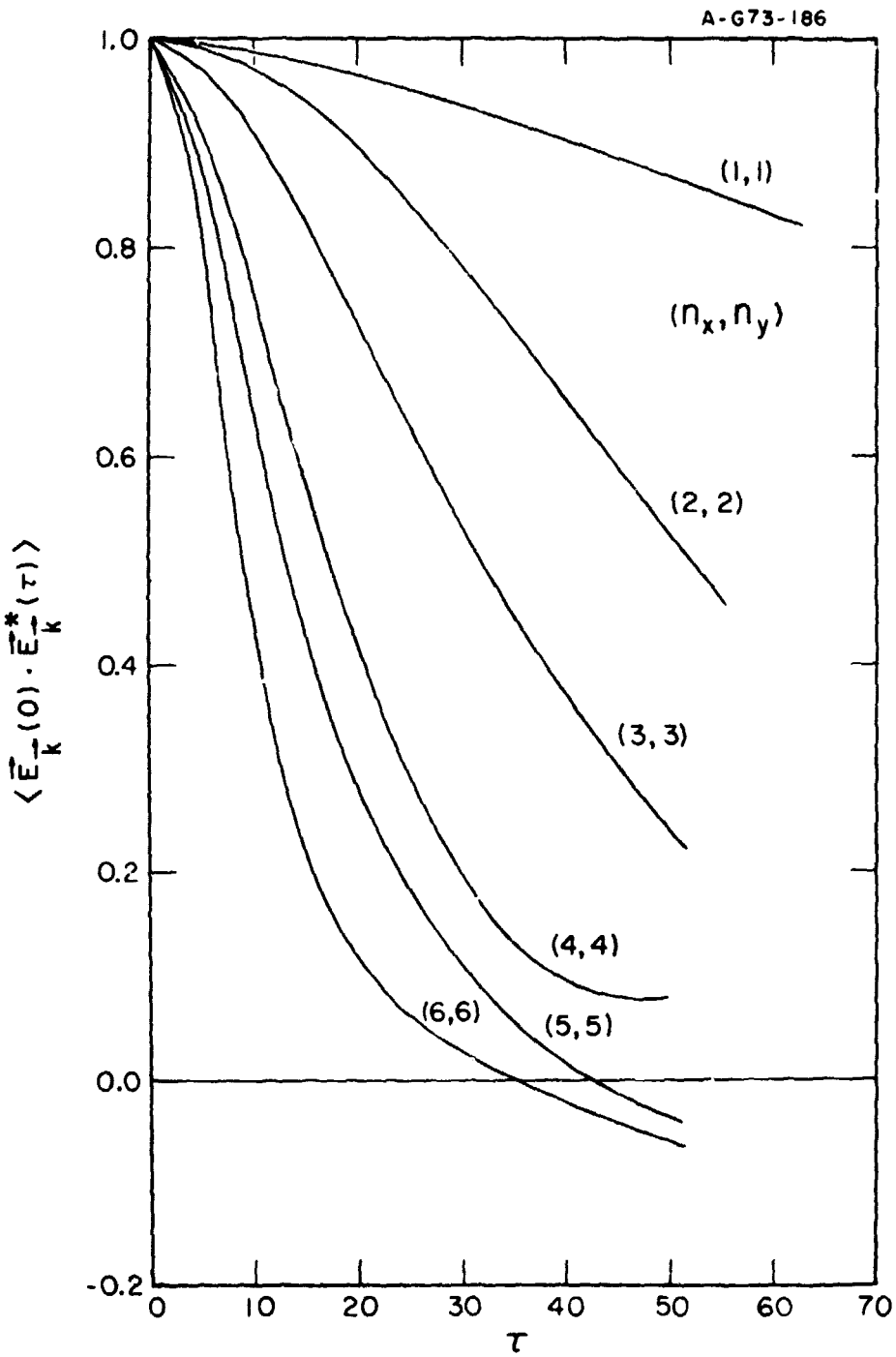

A-G73-195
$n_x = 0$
$n_y = 6$
EXP
T&T
T&M
$\langle \vec{E}_k(0) \cdot \vec{E}_k^*(\tau) \rangle$
τ
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0
10
20
30
40
50
60
70

Fig. 4b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0.6
0.4
0.2
0.0
-0.2
0
10
20
30
40
50
60
70
τ
$\langle \vec{E}_k(0) \cdot \vec{E}_k^*(\tau) \rangle$
(1, 1)
(n_x, n_y)
(2, 2)
(3, 3)
(4, 4)
(5, 5)
(6,6)

Fig. 5a

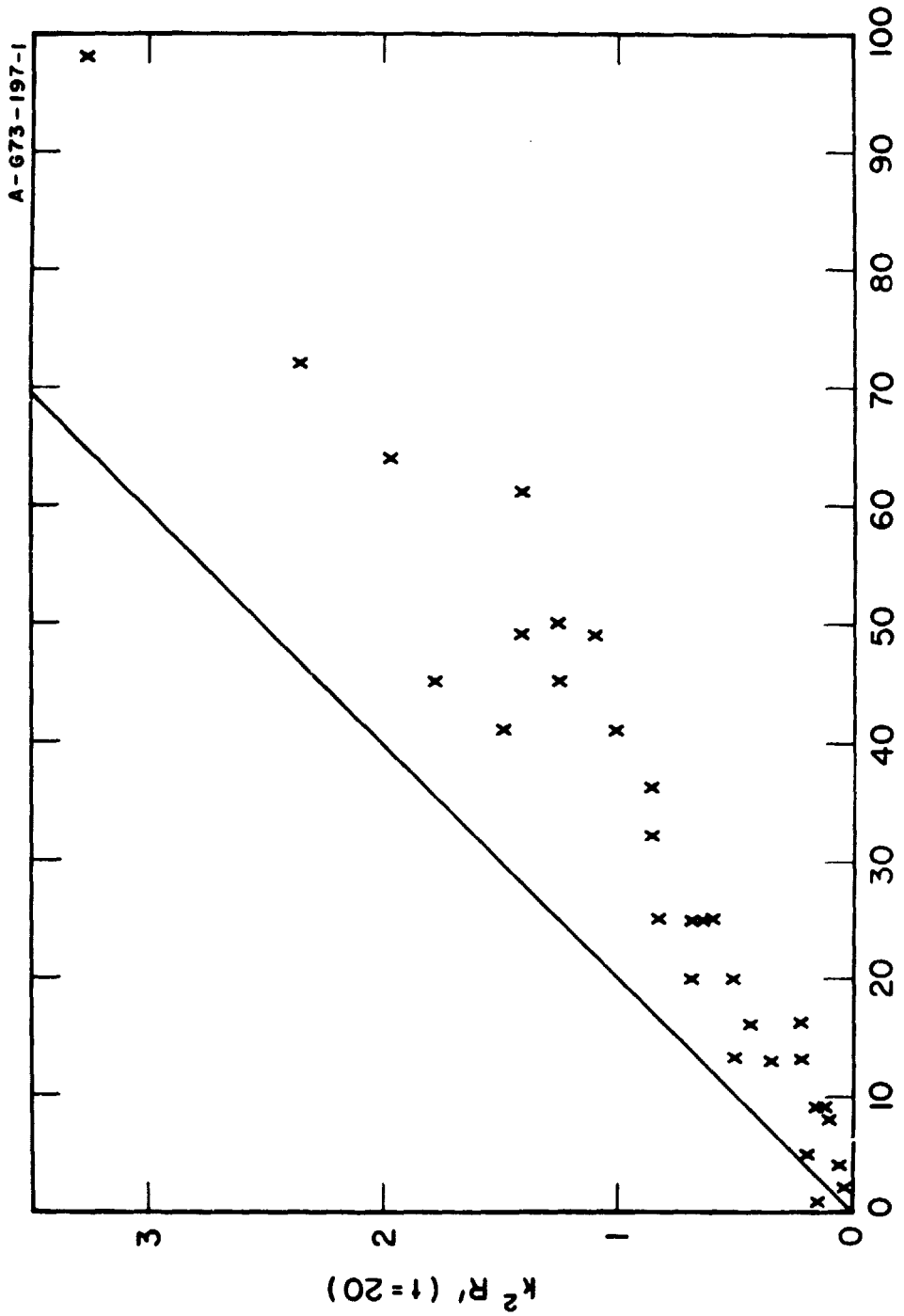

Fig. 5b